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Almost BPS black holes

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Abstract: We study non-BPS black hole solutions to ungauged supergravity with 8 supercharges coupled to vector multiplets in four and five dimensions. We identify a large class of five dimensional non-BPS solutions, which we call "almost BPS", that are supersymmetric on local patches and satisfy a first order flow governed by harmonic functions. By dimensional reduction, they give rise to new non-BPS solutions in four dimensions. These solutions allow for some nontrivial asymptotic moduli and multiple centres, similar to their globally supersymmetric cousins. We explicitly discuss a single centre and a two centre example.

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Contents

1 Introduction and overview

BPS black hole attractors, [1–3], due to a high degree of supersymmetry, are in some sense, the simplest black holes one can study. Consequently, it is not surprising that much progress in understanding the physics of black holes has come from studying these objects. The next simplest case one could choose to study is that of non-BPS black attractors [4, 5]. Although these solutions break supersymmetry, they seem to generically have similar near-horizon bosonic symmetries to their BPS cousins [6]. While much can be learnt from the near horizon behaviour, an obvious drawback is that, features of the full solution that cannot survive the near horizon limit are beyond reach. Such features include, the existence and properties of an interpolating solution to an asymptotic space-time, multi-centred solutions and associated split attractor flows. Unfortunately, in the a priori absence of first order equations, finding the full flow is challenging.[1]

It is known that in four dimensional, $\mathcal{N} > 2$ supergravity there is a formal analytic continuation of charges that can transform certain BPS to non-BPS solutions [7]. This is related to the fact that, at the horizon, the corresponding solutions to the attractor equations, are related by a sign change. For BPS attractors, one can obtain the full interpolating solution, from the horizon to the boundary of space-time, by replacing the

[1] A frame work for addressing the question of the existence of the full flow in the absence of first order equations is laid out in [5].

charges in the attractor values of the scalars with harmonic functions [8–12]. At the level of two derivative gravity, these solutions satisfy first order flow equations [4, 13, 14]. By analytic continuation of the harmonic functions rather than just the charges, this procedure can be generalised, in certain cases, to non-BPS attractors [15]. However, these cases are far from being generic — for instance once one turns on asymptotic B-fields, which are encoded in certain asymptotic moduli, the trick of analytically continuing the harmonic functions does not work — another approach is needed. It has been found that non-BPS first-order flow equations can also be found using the so-called "fake superpotential" formalism [16–18]. Recently, the addition of nontrivial B-fields was considered in [18–20], using the fake superpotential formalism and direct calculation, resulting in more complicated solutions that indeed do not fit in the known analytic continuation scheme.

In this work, we develop a method to obtain non-BPS solutions from the BPS ones that extends the results mentioned in the previous paragraph. We focus on non-BPS black holes in four and five dimensional ungauged supergravity. For concreteness we use theories with 8 supercharges, but the core of our results can be easily applied to the theories with more supersymmetry. We extend the simple analytic continuation of charges to an associated conjugation of the five dimensional BPS equations that describe the full flow. Using this scheme, we find a subclass of non-BPS solutions which, except for global constraints, obey the five dimensional BPS equations. Consequently, we call these solutions and the equations they obey "almost BPS". Our scheme reduces to a sign change for the examples with trivial B-fields, as in the known analytically continued solutions [15], and furthermore includes the recent solutions of [18, 19] with nontrivial B-fields, as well as novel multi-centre solutions. This generalises previous results for first order flows of single centred non-BPS solutions [16–18], albeit from a different perspective. Furthermore, hither to mysterious features of these solutions can be understood as a consequence of their almost BPS nature.

The main technical advantage in five dimensions is the existence of two independent sets of BPS solutions when the base space is flat. For a more general base, only one of these sets is BPS according to the known classification [21, 22]. The main idea we use to construct the almost BPS solutions is to exploit the simple fact that any Gibbons-Hawking space can be trivialised into flat space in local patches. This allows us to replace the known BPS solution by its would-be-BPS partner on every patch. The result is a new class of solutions that are BPS locally but not globally. In particular, they are described by first order equations that can be solved in terms of arbitrary harmonic functions, very similar to the BPS case.

This construction implies that our solutions are constrained to charges that can be obtained by sign changes from BPS charge vectors, even though the full solutions are very different. Thus, several known features of more general non-BPS solutions, such as flat directions [19, 23–27] remain out of reach. Nevertheless, our solutions can be used as seeds to generate new ones through known dualities. It remains an open problem to investigate what subspace of the full non-BPS spectrum can be obtained in this way. In special cases with enough symmetries, the most general non-BPS solution can be generated in this way, as in [19, 20] for the case of the STU model.

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Restricting attention to solutions that can be conveniently reduced on a circle, we obtain four dimensional non-BPS solutions characterised by arbitrary harmonic functions that allow for some nontrivial asymptotic moduli and multiple centres. When taking the near horizon limit, our solutions reproduce the known four dimensional non-BPS attractors. From the five dimensional point of view, the near horizon geometry is a time-like fibration over flat space, which locally satisfies the BPS equations presented in the next section. This implies that the near horizon geometries of our solutions are solutions to the attractor equations. It also allows us to construct stable multi-attractors with the same total charge as single centre non-BPS attractors whose stability is unclear [7, 15]. The same situation is found at the asymptotically flat region, leading to severe constraints on the form of the mass formula.

The crucial property of being locally supersymmetric implies strong constraints on the behaviour of our solutions in the interpolating region as well. The most important is the existence of a first order flow for the scalars governed by the almost BPS equations. As far as we know, by U-duality, this includes all known cases of four dimensional non-BPS first order flows in $\mathcal{N} = 2, 4, 6, 8$ theories and extends it to the multi-centre case.

This paper is organised as follows. In section 2 we give some background material on five dimensional supergravity, the classification of its BPS solutions and its reduction to four dimensional $\mathcal{N} = 2$ supergravity. Then, in section 3 we discuss the doubling of BPS solutions for the case of flat base and derive the almost BPS equations for a non-flat base. Section 4 contains two particular examples that are interesting from a four dimensional point of view, a single centre and a two centre solution. We conclude in section 5 with some extra comments and directions for future study.

2 5D supergravity and dimensional reduction

In this section we include some background material and establish notation. First, we briefly discuss five dimensional ungauged supergravity coupled to vector multiplets and the classification of the supersymmetric solutions of this theory. Next, we consider the dimensional reduction to four dimensions that yields the $\mathcal{N} = 2$ supergravity we are interested in.

2.1 BPS solutions of five dimensional supergravity

We consider five dimensional ungauged supergravity coupled to n_v vector multiplets. Unless stated otherwise, we adopt the conventions of [22]. Using a positive signature metric, the bosonic action takes the form [28]:

$$S = \frac{1}{2\kappa_5}\int \sqrt{-g}\, d^5x \Big(R - \frac{1}{2} Q_{IJ} F^I_{\mu\nu} F^{J\mu\nu} - Q_{IJ} \partial_\mu X^I \partial^\mu X^J \\ - \frac{1}{24} C_{IJK} F^I_{\mu\nu} F^J_{\rho\sigma} A^K_\lambda \bar{\epsilon}^{\mu\nu\rho\sigma\lambda} \Big), \tag{2.1}$$

where $\bar{\epsilon}^{\mu\nu\rho\sigma\lambda}$ is the completely antisymmetric Levi-Civita *tensor* with $|\bar{\epsilon}^{\mu\nu\rho\sigma\lambda}| = 1/\sqrt{-g}$ (the orientation will be defined below). The indices, $I, J, K \ldots = 1, \ldots, n_v$, label the vector multiplets and C_{IJK} is a totally symmetric constant tensor. The scalars X^I satisfy the constraint:

$$X_I X^I = 1\,, \qquad X_I \equiv \frac{1}{6} C_{IJK} X^J X^K\,, \tag{2.2}$$

so that only $n_v - 1$ of them are independent. We denote the independent scalars as, χ^a, where $a = 1, \ldots, n_v - 1$. The scalar dependent coupling matrix, Q_{IJ}, has the form:

$$Q_{IJ} = \frac{9}{2} X_I X_J - \frac{1}{2} C_{IJK} X^K\ . \tag{2.3}$$

For simplicity, we will restrict our attention to the case where the scalars take values in a symmetric space. One can then use the identities:

$$\frac{4}{3}\delta_{I(L} C_{MPQ)} = C_{IJK} C_{J'(LM} C_{PQ)K'} \delta^{JJ'} \delta^{KK'}\,, \tag{2.4}$$

$$X^I = \frac{9}{2} C^{IJK} X_J X_K\,, \tag{2.5}$$

where $C^{IJK} \equiv \delta^{II'} \delta^{JJ'} \delta^{KK'} C_{I'J'K'}$ and the constraint in (2.2) was used in obtaining (2.5).

The supersymmetric solutions of this theory have been classified in [21, 22] and fall in two distinct classes, depending on whether the vector, $\bar{\epsilon}\gamma^\mu\epsilon$, constructed from the Killing spinor, ϵ, is time-like or null. We will restrict our attention to the time-like case, in which the BPS conditions imply that the metric is a time-like fibration over a hyper-Kähler base space, which we briefly summarise below.

Given a hyper-Kähler 4-manifold with a metric, h_{mn}, we choose the associated complex structures, $X^{(i)}$, as anti-self-dual and assume for the moment that they are unique. Given these data, the metric and gauge fields of a supersymmetric solution can be written locally as:

$$\begin{aligned} ds^2 &= -f^2(dt + \omega)^2 + f^{-1} h_{mn} dx^m dx^n \\ F^I &= d(X^I e^0) + \Lambda^I. \end{aligned} \tag{2.6}$$

Here, $e^0 = f(dt + \omega)$, the indices, $m, n, \ldots = 1, \ldots, 4$, label coordinates on the base, the $\Lambda^I(x^m)$ are arbitrary closed self-dual forms on the base, and $f > 0$, is assumed to be a globally defined function. A positive orientation is chosen using $e^0 \wedge \eta$ as the volume form, where η is a positive orientation on the base manifold.

Once the Λ^I's are chosen, the function f and the X^I are determined by solving a Poisson equation on the base:

$$\Delta\left(f^{-1} X_I\right) = \frac{1}{12} C_{IJK} \Lambda^J_{mn} \Lambda^{mn\,K}\,, \tag{2.7}$$

where Δ is the Laplacian on the hyper-Kähler manifold. Finally, the one-form, ω, is determined by solving:

$$f d\omega = G = G^+ + G^-\,, \qquad X_I \Lambda^I = -\frac{2}{3} G^+\,, \tag{2.8}$$

where $G^\pm$ are self- and anti-self-dual forms on the hyper-Kähler base. These BPS solutions are invariant under an SU(2) subgroup of the base space isometry group and, in an orthonormal basis, their respective Killing spinors satisfy $\gamma^0\epsilon = \mathrm{i}\epsilon$. Note that if our assumption of the uniqueness of the complex structures is not true, there is extra freedom on the BPS solutions that can be written down, changing the above equations. This will be seen explicitly in the following, but for the moment we continue to assume uniqueness.

In this work we are interested in solutions with a four dimensional interpretation, so we will demand that the base space has a compact isometry along which we can perform dimensional reduction. Under the assumption that the associated Killing vector is tri-holomorphic (i.e. leaves the complex structures invariant), and is a symmetry of the full solution, the base space can only be a Gibbons-Hawking space [29]. In this case the above equations simplify enough to be solved explicitly [22]. For a Gibbons-Hawking space [30], which is itself a fibration over a flat Euclidean base, the metric, h_{mn}, can be written:

$$h_{mn}dx^m dx^n = H^{-1}\left(d\psi + \chi_i dx^i\right)^2 + H\delta_{ij}dx^i dx^j, \tag{2.9}$$

$$\nabla \times \boldsymbol{\chi} = \nabla H. \tag{2.10}$$

Here, ∇ is the standard vector derivative on the Euclidean 3-space, $\mathbb{R}^3$, with coordinates x^i, $i, j = 1, 2, 3$, $H(x^i)$ is a harmonic function on $\mathbb{R}^3$ and $0 \leq \psi \leq 4\pi$. The isometry group is $\mathrm{SU}(2) \times \mathrm{U}(1)$, where the $\mathrm{U}(1)$ is generated by the Killing vector $\partial/\partial\psi$. We will consider the specific examples of flat space ($H = 1$ or $H = 1/|\mathbf{x}|$) and Taub-NUT space ($H = h^0 + p^0/|\mathbf{x}|$).

The complex structures associated with (2.9) are given by

$$X^{(i)} = \left(d\psi + \chi_j dx^j\right) \wedge dx^i - \frac{1}{2}H\epsilon_{ijk}dx^j \wedge dx^k. \tag{2.11}$$

Using (2.10), one can easily see $dX^{(i)} = 0$. Imposing their anti-self-duality fixes the orientation of the base space so that the volume form is

$$H d\psi \wedge dx^1 \wedge dx^2 \wedge dx^3. \tag{2.12}$$

The complex structures (2.11) are globally defined and, in all but one case, unique. The exception is flat space (in coordinates such that $H = 1$, $\chi = 0$), in which case one can also choose the opposite relative sign in (2.11) and the forms, $X^{(i)}$, remain closed. It follows that for flat space there are two triplets of complex structures: one self-dual and one anti-self-dual. This observation implies an enlargement of the set of BPS solutions in that case, to be discussed in section 3.

The explicit BPS solution for a Gibbons-Hawking base can be described in terms of H and an additional $2n_v+1$ harmonic functions [22]. The self-dual forms Λ^I, defined in (2.6), can be written as:

$$\Lambda^I = -\frac{1}{2}(d\psi + \chi) \wedge (W^I{}_j)dx^j - \frac{1}{4}H\epsilon_{ijk}(W^I{}_k)dx^i \wedge dx^j\,, \qquad W^I{}_i = \partial_i\left(\frac{K^I}{H}\right)\,, \tag{2.13}$$

where the K^I are arbitrary harmonic functions related to the magnetic charges of the solution. Given these functions, the rest of the solution is found by solving

$$f^{-1}X_I = \frac{1}{24}H^{-1}C_{IJK}K^J K^K + L_I\,, \tag{2.14}$$

$$\omega_5 = -\frac{1}{48}H^{-2}C_{IJK}K^I K^J K^K - \frac{3}{4}H^{-1}L_I K^I + M \tag{2.15}$$

$$\nabla\times\hat{\omega} = H\nabla M - M\nabla H + \frac{3}{4}(L_I\nabla K^I - K^I\nabla L_I)\,. \tag{2.16}$$

Here, the one-form ω is decomposed as

$$\omega_m dx^m = \hat{\omega}_i dx^i + \omega_5(d\psi + \chi)\,, \tag{2.17}$$

and L_I, M are arbitrary harmonic functions associated with the electric charges and the angular momentum along the ψ direction respectively.

2.2 4D/5D connection

In the following, we will be interested in the properties of the solutions to the four dimensional theory obtained by reducing the theory (2.1) on a circle. In order to make the connection clear, we present some of the relevant formulae for this reduction following [31, 32], but using slightly different conventions.

The relevant ansatz for the reduction is written as:

$$\begin{aligned} ds^2 &= e^{2\phi}\,ds^2_{(4)} + e^{-4\phi}\,(d\psi - A^0_{(4)})^2\,, \\ A^I &= A^I_{(4)} + C^I\,(d\psi - A^0_{(4)})\,, \\ \hat{X}^I &= e^{-2\phi}\,X^I\,, \end{aligned} \tag{2.18}$$

where $ds^2_{(4)}$ is the four dimensional line element and the C^I, $\hat{X}^I$ will make up the four dimensional complex scalars. Here, the coordinate along the circle, ψ, runs over $0 \le \psi \le 4\pi$. This fixes the four dimensional Newton constant: $G_4 = G_5/4\pi$.

Reducing the action in (2.1) along a circle [32], one obtains a four dimensional $\mathcal{N}=2$ supergravity characterised by the pre-potential:

$$F(Y) = -\frac{1}{12}\,\frac{C_{IJK}\,Y^I Y^J Y^K}{Y^0}\,, \tag{2.19}$$

where the Y^A are n_v+1 complex scalar fields ($A = 0, I$). These scalars are also subject to a constraint that makes one of them redundant. We solve it by working throughout in special coordinates:

$$z^I = \frac{Y^I}{Y^0} = C^I + \mathrm{i}\hat{X}^I\,. \tag{2.20}$$

The explicit four dimensional bosonic action that results from the reduction is [33, 34]:

$$\begin{aligned} S_4 = \frac{1}{2\kappa_4}\int d^4x\sqrt{-g}\Big(R - 2g_{I\bar{J}}\partial_\mu z^I\partial^\mu\bar{z}^J + \frac{1}{2}\mathrm{Im}\,\mathcal{N}_{AB}F^A_{\mu\nu}F^{B\mu\nu} \\ -\frac{1}{2}\mathrm{Re}\,\mathcal{N}_{AB}F^A_{\mu\nu} * F^{B\mu\nu}\Big), \end{aligned} \tag{2.21}$$

where $*F^{A\,\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F^A_{\rho\sigma}$ with $\epsilon^{0123} = 1/\sqrt{-g}$. The four-dimensional gauge couplings, $\mathcal{N}_{AB}$, are given by

$$\mathcal{N}_{AB} = \bar{F}_{AB} + 2\,\mathrm{i}\,\frac{\operatorname{Im} F_{AC} \operatorname{Im} F_{BD}\, Y^C Y^D}{\operatorname{Im} F_{FE}\, Y^F Y^E}\,, \tag{2.22}$$

where $F_A = \partial F/\partial Y^A\,,\ F_{AB} = \partial^2 F/\partial Y^A \partial Y^B$. Finally, the moduli space metric, $g_{I\bar{J}}$, is:

$$g_{I\bar{J}} = \frac{\partial}{\partial z^I}\frac{\partial}{\partial \bar{z}^J}K\,, \qquad \mathrm{e}^{-K(z,\bar{z})} = \frac{\mathrm{i}}{12}C_{IJK}(z^I - \bar{z}^I)(z^J - \bar{z}^J)(z^K - \bar{z}^K)\,. \tag{2.23}$$

Note that there is an extra gauge multiplet in four dimensions, represented by the zeroth index, which comes from the reduction of the five dimensional graviton supermultiplet. The scalars are complexified in the reduction, with the n_v pseudoscalars C^I coming from the gauge fields paired with the n_v physical scalars X^I and the Kaluza-Klein scalar ϕ as in (2.20). In string theory language, the C^I are interpreted as components of the B-field reduced to four dimensions.

In contrast to the above, finding the explicit relation between the charges of a solution to the five dimensional theory and its four dimensional reduction can be subtle, especially in the presence of magnetic charges. Since, in the following, we are interested in the four dimensional interpretation, we will be dealing only with four dimensional electric and magnetic charges, using the natural formulae:

$$q_A \equiv (q_0\,, q_I) = \frac{1}{4\pi}\int_{S^2}\left(\operatorname{Im}\mathcal{N}_{AB} *F^B - \operatorname{Re}\mathcal{N}_{AB}F^B\right)\,, \tag{2.24}$$

$$p^A \equiv (p^0\,, p^I) = \frac{1}{4\pi}\int_{S^2} F^A\,, \tag{2.25}$$

where the sphere encloses the region of interest.

This completes the dictionary between the solutions of (2.1) and (2.21) in the general case. For our purposes though, we will be interested only in five dimensional solutions that can be written as a time-like fibration over a Gibbons-Hawking base space as in (2.6) and (2.9). Under this restriction, there are significant simplifications. In particular, the Kaluza-Klein scalar, ϕ, and the four dimensional metric, are written as:

$$ds^2_{(4)} = -\mathrm{e}^{2U}(dt + \hat{\omega}_i\,dx^i)^2 + \mathrm{e}^{-2U}d\vec{x}^2 \qquad e^{-4U} = \frac{H^2}{f^2}e^{-4\phi} = H\,f^{-3} - (H\,\omega_5)^2\,. \tag{2.26}$$

Moreover, the Kaluza-Klein gauge potential seen in four dimensions is given by the expression:

$$A^0_{(4)} = \omega_5\,H^2\,\mathrm{e}^{4U}\,(dt + \hat{\omega}_i\,dx^i) - \chi_i\,dx^i\,, \tag{2.27}$$

where the notation in (2.17) is used for ω. In the following, we will use the above formulae both explicitly and implicitly, to give a four dimensional interpretation to our results. More explicit details can be found in.[2]

[2] Note that our tensor C_{IJK} is related by a factor of two to the one in [32].

3 BPS and almost BPS solutions

3.1 BPS solutions with flat base space

As noted in section 2.1, for the special case of a flat base space there is some extra freedom in choosing the hyper-Kähler complex structures: namely they can be chosen to be either self- or anti-self-dual. This is a result of the larger group of rotations for $\mathbb{R}^4$, namely $\mathrm{SO}(4) \equiv \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. The two triplets of complex structures rotate under different linear combinations of $\mathrm{SU}(2)_R$ and $\mathrm{SU}(2)_L$.

In this special context, there is a second supersymmetric solution one can write down, in which all the fields are left invariant under the self-dual complex structures [21]. Explicitly, the relevant BPS conditions (2.6)–(2.8) for the flat case are modified to:

$$ds^2 = -f^2(dt+\omega)^2 + f^{-1}h_{mn}dx^m dx^n \,, \tag{3.1}$$

$$F^I = \pm\Big(d(X^I e^0) + \Lambda^I\Big)\,, \tag{3.2}$$

$$X_I\Lambda^I = -\frac{2}{3}G^{\pm}\,, \tag{3.3}$$

$$\Delta\left(f^{-1}X_I\right) = \frac{1}{12}C_{IJK}\Lambda^J_{mn}\Lambda^{mn\,K}\,. \tag{3.4}$$

Here, Λ^I are self-dual forms on the base for the upper sign and anti-self-dual for the lower sign. These pairs of BPS solutions are aligned with the two complex structures in the sense that they are invariant under the corresponding SU(2) subgroups of the SO(4) isometry group and, in an orthonormal basis, their respective Killing spinors satisfy $\gamma^0\epsilon = \pm i\epsilon$. Some examples of supersymmetric pairs were written down in [21], using right- and left-invariant one-forms on S^3.

An interesting property of these BPS pairs is that when reducing to four dimensions along a circle, it is not possible to retain both of them in the BPS spectrum. This is because any reduction ansatz can only respect one $\mathrm{SU}(2) \equiv \mathrm{SO}(3)$ isometry, while the two sets of BPS solutions respect a different SU(2) in five dimensions. Thus, choosing a particular SU(2) to keep in the reduction, the other one is broken and all the BPS solutions aligned with it are lost as well. Equivalently, the Killing spinors in one set of solutions are invariant under the SU(2) that is being broken, while those of the other set are not. This means that under the reduction, the Killing spinors in one set will be trivially reduced, but the ones in the other set will be charged under the Kaluza-Klein U(1), violating the natural assumption of invariance and leading to a non-BPS solution. A very similar situation was encountered in [35, 36], where it was shown that if one considers the full Kaluza-Klein tower, supersymmetry is recovered.

This "supersymmetry without supersymmetry" effect [36], was observed in [32, 37], in the context of lifting to five dimensions the near horizon geometry of a four dimensional non-BPS black hole, which can be written as a solution with flat base space. In the particular electric case discussed there, one has $\Lambda^I = 0$, $\omega = 0$ and $H = p^0/r$ for flat base space with a conical singularity (the coordinate ψ has a fixed range as in (2.9)). Then the two five dimensional BPS solutions above are identical with respect to the metric and the scalars, but have opposite electric charges. The explicit solutions are described by (in our conventions):

$$\begin{aligned} ds^2 &= -f^2 dt^2 + f^{-1}\left(\frac{p^0}{r}dx^i dx^i + \frac{r}{p^0}(d\psi + p^0\cos\theta\phi)^2\right) \\ F^I &= \pm d(fX^I)\wedge dt, \qquad f^{-1}X_I = L_I = \frac{2}{3}\frac{q_I}{r}. \end{aligned} \tag{3.5}$$

Upon reduction to four dimensions preserving the SO(3) symmetry of the directions x^i, they give the BPS and the non-BPS attractor discussed in [32, 37] for the plus and minus sign respectively.[3] From the four dimensional point of view the two solutions are related by analytic continuation of the charges [7].

It should be noted that this observation does not affect the 4D/5D connection for BPS solutions as described in [38, 39]. There, a Taub-NUT base that interpolates between a five dimensional and a four dimensional solution was used to argue that the BPS index is the same in four and five dimensions. As Taub-NUT has a unique triplet of complex structures, there is only one BPS solution for each choice of harmonic functions. Thus, all asymptotically Taub-NUT BPS solutions in five dimensions are mapped to asymptotically flat BPS solutions with appropriate charges, under dimensional reduction [31]. The same holds for any Gibbons-Hawking space except flat space.

This is explicitly seen from the asymptotically Taub-NUT extensions of the two attractor solutions. The solution with the plus sign in (3.5) is compatible with the anti-self-dual complex structures of $\mathbb{R}^4$ and its asymptotically Taub-NUT extension is the BMPV black hole [40] in the centre of Taub-NUT. In view of the anti-self-duality of the complex structures of this base, it is a BPS solution [21]. The solution with the minus sign is instead compatible with the self-dual complex structures of $\mathbb{R}^4$. Its Taub-NUT extension was constructed in [18], and is non-BPS. In the following we rederive this solution and clarify the origin of its special properties, as part of a general technique of constructing non-BPS solutions.

3.2 Almost BPS solutions

In the less restrictive case of a general Gibbons-Hawking space, one might wonder about the fate of the two different supersymmetric solutions. This becomes especially interesting in view of the fact that both kinds of BPS solutions with a flat base can be the near horizon region of (not necessarily BPS) black holes embedded in a more general space, as mentioned above. The existence of full interpolating Taub-NUT solutions for the two attractors above, only one of which is BPS, suggests that the two different solutions might survive as BPS/non-BPS pairs in this case.

Indeed, it is straightforward to show that even if the base is not flat, both expressions in (3.1)–(3.3) solve the equations of motion. An outline of this calculation can be found in appendix A. For a general base, the anti-self-duality of the hyper-Kähler structures allows only for the one with the upper sign to be supersymmetric, whereas the other one is not.

[3] If the other SO(3) symmetry of $\mathbb{R}^4$ is chosen in the ansatz, one ends up with the same solutions in four dimensions, but their origins in five dimensions are interchanged.

An intuitive picture of the relation between the two solutions can be given for a Gibbons-Hawking base space. The base space is a U(1) bundle over $\mathbb{R}^3$, so it can be trivialised into $\mathbb{R}^4$ by a suitable choice of coordinates on any local patch. One then has a choice between self-dual or anti-self-dual complex structures on every such patch as before, so that both expressions in (3.1)–(3.3) constitute BPS solutions.

By extending to the full base space, only the anti-self-dual structures on local patches can be integrated to the unique global complex structures (2.11). In contrast, the local self-dual structures can be integrated to the almost hyper-Kähler structures:

$$\tilde{X}^{(i)} = \left(d\psi + \chi_j dx^j\right) \wedge dx^i + \frac{1}{2} H \epsilon_{ijk} dx^j \wedge dx^k, \tag{3.6}$$

that are globally defined, but not integrable: $d\,\tilde{X}^{(i)} \neq 0$. The existence of the forms (3.6) allows one to construct globally defined fields by aligning local solutions on every patch with the appropriate restriction of these structures.[4] It is then clear why both signs in (3.1) provide a solution to the equations of motion, since they can be viewed as constructed locally from BPS solutions aligned with the forms in (3.6). The difference is that the one with the upper signs is compatible with the global complex structures and is a global BPS solution. The second solution fails to be supersymmetric only due to a global obstruction, providing an example of a nonsupersymmetric solution with the peculiar property of admitting four supercharges on local patches. In fact, it is expected to have all the local properties of a BPS solution, which are behind most of the computational simplifications in that case. This property, which is based on the existence of an almost hyper-Kähler structure, motivates the nickname, "almost BPS", for these solutions. By the same argument, our non-BPS solutions are supersymmetric on local patches for a hyper-Kähler base more general than Gibbons-Hawking if there exists a globally defined almost hyper-Kähler structure.

Here, we will restrict to Gibbons-Hawking base spaces for simplicity. In this case, the almost BPS solutions can be specified through arbitrary harmonic functions as for the BPS case, following the algorithm in [22]. First write

$$\omega = \omega_5 \left(d\psi + \chi_i dx^i\right) + \hat{\omega}_i dx^i, \tag{3.7}$$

$$\Lambda^I = \frac{1}{2} A^I_i \left(d\psi + \chi_j dx^j\right) \wedge dx^i - \frac{1}{4} H \epsilon_{ijk} A^I_k dx^i \wedge dx^j, \tag{3.8}$$

for the one-form ω and the anti-self-dual form in (3.1), where A^I are arbitrary one-forms to be determined.

Using (2.9), closure of Λ^I reduces to the relations:

$$\nabla \times \mathbf{A}^I = 0\,, \qquad \nabla \cdot \mathbf{A}^I = 0\,, \tag{3.9}$$

which in turn imply that locally:

$$\mathbf{A}^I = \nabla K^I\,, \tag{3.10}$$

[4] Note that the restriction of these forms on a patch is transformed to a constant by the coordinate transformation that trivialises the patch. It is the noncompatibility of these local coordinate transformations that makes the global forms not integrable. This is also what prohibits the existence of a corresponding global Killing spinor.

for some harmonic functions K^I. The equation for $f^{-1}X_I$ in (3.3) reduces to

$$\nabla^2\left(f^{-1}X_I\right) = \frac{1}{12}HC_{IJK}\nabla K^J\nabla K^K = \frac{1}{24}H\nabla^2\left(C_{IJK}K^JK^K\right)\,, \tag{3.11}$$

which can be solved up to a set of arbitrary harmonic functions L_I, given the K^I. Here, we will restrict to solutions of the slightly stronger relation:

$$\nabla\left(f^{-1}X_I\right) = \frac{1}{24}\Big(H\nabla\left(C_{IJK}K^JK^K\right) - C_{IJK}K^JK^K\nabla H\Big) + \nabla L_I\,, \tag{3.12}$$

even though there might be physically interesting solutions not captured by it. The advantage of this simplification is that the scalars are governed by a first order flow very similar to the BPS one.

Finally, we find the conditions on ω_5 and $\hat{\omega}_i$. Writing out the first equation in (3.3) using (2.8) and (3.10) one gets:

$$\nabla\times\hat{\omega} + \nabla\left(H\omega_5\right) = \frac{3}{2}f^{-1}HX_I\nabla K^I \tag{3.13}$$

Taking the divergence of this gives the integrability condition

$$\nabla^2\left(H\omega_5\right) = \frac{3}{2}\nabla\left(f^{-1}HX_I\right)\cdot\nabla K^I, \tag{3.14}$$

which can be solved up to an arbitrary harmonic function M, given the solution of (3.12). Substituting in (3.13), the one-form $\hat{\omega}$ can be determined up to a total derivative (removable by a change of coordinates).

Observe that, just like its BPS partner, an almost BPS solution is determined by $2n_v+2$ harmonic functions H, K^I, L_I, M, which encode the charges as in section 2.1. Note however, that if H is such that the base is flat, the BPS/almost BPS pairs degenerate into the supersymmetric pairs in the previous section. Hence, there are no asymptotically flat five dimensional nonsupersymmetric solutions in the class described here. On the other hand there are solutions that asymptote to $\mathbb{R}^3\times S^1$, allowing for an interpretation as asymptotically flat solutions in four dimensions. That is the case of interest in this work.

In any case, the almost BPS solutions are a five dimensional analogue of the four dimensional non-BPS black holes that can be obtained from BPS solutions through a change of relative signs in charges (and harmonic functions). Comparing (3.11)–(3.14) with (2.14)–(2.16), shows that they are related in that way if $K^I=0$, but the general case is much more complicated than a change of relative signs, as it involves different powers of the harmonic function H. For the special case of flat base, where one can choose $H=1$, the supersymmetric pairs are always related by such a sign change, but not if one takes $H=1/r$. The latter choice typically leads to genuinely different BPS solutions that are not asymptotically flat, as in [21].

Interestingly, by reducing along the $\partial/\partial\psi$ direction, our solutions give rise to nonsupersymmetric solutions of four dimensional $\mathcal{N}=2$ supergravity that are more general in several ways than the ones produced by the four dimensional sign change. In particular, the five dimensional almost BPS solutions allow for some nontrivial moduli at infinity as

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in [18, 19] and multiple centres, unlike the simple flip of signs in four dimensions that only works for single centre solutions without B-fields.

Despite this, our solutions are far from being the most general non-BPS black holes, as they represent very special points in the duality orbits. For example, we cannot construct a $D0$–$D6$ non-BPS black hole (see e.g [19, 20, 32]) using the almost BPS equations, as they were derived from the BPS conditions, that do not allow for such an object. Moreover, it is known that non-BPS black holes generically exhibit flat directions all along their flow in both four and five dimensions [23–27]. This feature is also not captured by our almost BPS solutions, since the BPS solutions have no flat directions either. Nevertheless, our solutions can be used as seed solutions to generate new ones through four dimensional dualities. If there are enough symmetries in a specific theory, the most general non-BPS solution can be generated in this way, as in [19, 20] for the case of the STU model.

Unfortunately though, the explicit equations are significantly more complicated than in the BPS case and can not be solved formally as in [22] in the most general case. Nevertheless, the full interpolating solution for any specific choice of harmonic functions should be readily obtainable by a numerical approach, since the solution is determined through Poisson equations which are well studied. The rest of this paper is devoted to analysing examples of almost BPS solutions and their properties.

4 Examples of almost BPS solutions

We now turn to some examples of almost BPS solutions in Taub-NUT. We discuss a single centred and a two centred example, both of which are very interesting from a four dimensional perspective. In doing this, we draw on intuition from the study of the known BPS partners of these solutions. The two cases are largely independent, but share some common features that are generic for our solutions and follow from their construction.

The most important feature is the special structure of their near horizon and asymptotic regions. Indeed, since the near horizon region of our solutions can be written as a timelike fibration over flat space, it preserves four supercharges by construction. In fact, it preserves all eight supercharges, since the near horizon region is maximally supersymmetric for BPS solutions. The same holds trivially for the asymptotic region in our Taub-NUT examples up to compactness of one coordinate, as they asymptote to $\mathbb{R}^3 \times S^1$. This property simplifies the near horizon analysis, but more importantly implies that the mass of these solutions takes the same form as for its BPS partner, namely that it is a simple sum of the charges. This suggests that the constituents are marginally bound. This feature was observed for nonsupersymmetric solutions of the STU model in four dimensions [19], some of which can be lifted to almost BPS solutions in five dimensions, as we will show below. Note that the BPS nature of the attractors and the asymptotic region of our solutions is invisible from a four dimensional perspective, as explained in section 3.1.

A second, more intricate, property that also follows from the almost BPS conditions is the existence of a first order flow for the scalars as in [16–18]. In fact the function governing this flow can be obtained from the BPS central charge, that plays this role in the BPS case, using our recipe. The above statements apply both to the five dimensional and the four dimensional theory, by dimensional reduction. In the single centred case we will show such an explicit example, reinterpreting the known solution of [18] in our framework.

Finally, a technical point is that our equations allow for at least one extra asymptotic modulus than what can be obtained through the four dimensional sign flip. It is described by the constant part of the arbitrary harmonic function M in the solution of (3.14). This constant is always left undetermined because only derivatives of $H\omega_5$ and thus M appear in all expressions. This is a crucial difference from the BPS equation (2.16), through which the constant part of M is related to other quantities when cancellation of Dirac-Misner strings is imposed. From a five dimensional point of view, this constant is interpreted as rotation of the asymptotic region, as in [18, 41]. From the point of view of the four dimensional theory it is seen as an asymptotic B-field, which allows us to make contact with the purely four dimensional derivation of [19] in the single centre case.

4.1 Non-BPS electric black hole

In this subsection, we make contact with the single centre solutions in [18, 19]. Since these solutions were thoroughly examined in [18], we will be brief, emphasising general aspects. Similar to the BMPV solution on the BPS side, we choose the harmonic functions as:

$$H = h^0 + \frac{p^0}{r}\,, \quad K^I = 0\,, \quad L_I = l_I + \frac{2}{3}\frac{q_I}{r}\,, \quad M = -b\,, \tag{4.1}$$

where $l_I\,, b$ are constants that will turn out to be related to asymptotic moduli and $p^0\,, q_I$ are the Taub-NUT charge and the electric charges of the solution, as computed in the four dimensional theory. Then (3.11)–(3.14) are solved by:

$$f^{-1}X_I = L_I\,, \quad \Rightarrow \quad f^{-3} = \frac{9}{2}C^{IJK}L_IL_JL_K\,, \quad X^I = \frac{9}{2}f^2\,C^{IJK}L_JL_K \tag{4.2}$$

$$\omega_5 = -\frac{b}{H}\,, \qquad\qquad \hat{\omega} = 0\,. \tag{4.3}$$

As mentioned in section 2.1, we consider the case of a symmetric scalar manifold which allows us to solve the scalar equation explicitly. These expressions agree with the ones in [18] and by using (3.1),(3.8), the full non-BPS solution is reproduced, with the extra requirement of no Dirac-Misner strings. Note that the extra rotation b is invisible in the near horizon limit, which is described by the supersymmetric attractor in (3.5), as expected. In the following we concentrate on the four dimensional interpretation of this solution.

When reduced to four dimensions using (2.18), the metric is the static limit of (2.26), with

$$e^{-4U} = \frac{9}{2}HC^{IJK}L_IL_JL_K - b^2\,, \qquad \lim_{r\to\infty} e^{-4U} = h^0l^3 - b^2 = 1, \tag{4.4}$$

$$l^3 = l^Il_I, \qquad l^I = \frac{9}{2}C^{IJK}l_Jl_K\,, \tag{4.5}$$

where we imposed four dimensional asymptotic flatness and introduced some useful notation. The four dimensional scalars (2.20) and their asymptotic values are given by:

$$z^I = \frac{f\,X^I}{H}\left(b + \mathrm{i}e^{-2U}\right) \quad \Rightarrow \quad z^I_\infty \equiv x^I + \mathrm{i}y^I = \frac{l^I}{h^0l^3}(b+i)\,. \tag{4.6}$$

Thus, the parameter b is identified as an asymptotic B-field in the four dimensional theory.

The ADM mass associated to this four dimensional solution is simply found by expanding the metric to first order in $1/r$. The result is:

$$M_{ADM} = \frac{1}{4G_4}\left(p^0 l^3 + 2\,h^0\,l^I q_I\right) = \frac{\sqrt{1+b^2}}{4G_4}\left((1+b^2)y^{3/2}p^0 + \frac{2\,y^I q_I}{y^{3/2}}\right). \tag{4.7}$$

Here we replaced the l_I by the four dimensional asymptotic scalars y^I, using the constraint in (4.4), as well as:

$$y^{3/2} = \sqrt{\frac{1}{6}C_{IJK}y^I y^J y^k}, \qquad l^3 = \sqrt{\frac{1}{6}C_{IJK}l^I l^J l^k}\,, \tag{4.8}$$

where the second relation is an identity following from (2.4). This mass formula is simply a sum of charges just like its BPS partner as anticipated above. It agrees with the expression derived in [20] for the special case of the STU model.

In fact, the seed solution of [19], which was argued to have the minimal set of parameters required to describe any non-BPS black hole in the STU model up to U-duality is dual to the above solution for the STU case. More generally, the solutions above have half of the charges turned on and asymptotic moduli as in (4.6) for any scalar manifold. Based on this, we expect that the most general non-BPS black hole for a four dimensional $\mathcal{N} = 2$ theory with a symmetric scalar manifold can be obtained by applying dualities on the above solution, along the lines of [20]. For more general scalar manifolds, one can still apply a restricted set of dualities to generate solutions, like the electric-magnetic duality in [18], but a case by case analysis is required.

A further property of this solution is that it is described by the first order formalism of [16–18], as shown explicitly in [18]. In particular, the so called fake superpotential describing it can be obtained from the BPS central charge using our recipe. In five dimensions, this amounts to a sign change for the electric charges, while the four dimensional flow is governed by a function very similar to the central charge. It follows that all non-BPS solutions that can be obtained by U-duality from the above solution, must also exhibit this property. Again, for symmetric scalar manifolds this can be done explicitly, following [20].

Finally, we discuss two possible generalisations. The first is to the multi-centre case, by adding more centres in the harmonic functions $H\,, L_I$ in (4.1), generalising the base space to multi-Taub-NUT. Upon reduction to four dimensions, these solutions seem to agree qualitatively with the solutions of [42], though we have not considered them in detail. A second generalisation would be to turn on the magnetic harmonic functions. It is important to notice that in the BPS case this would be irrelevant due to the fact that only ratios of the form K^I/H appear in the BPS conditions, as observed in [43]. In contrast, our equations (3.12)–(3.14) do not appear to have such an invariance. In this case, the functions $f\,, \omega_5$ diverge at the centre[5] as r^{-3} and r^{-4} respectively, making them unattractive at first sight. However, the near horizon geometry of these solutions is described by the

[5] One would naively expect to find a horizon at $r = 0$ in these coordinates, as in all other cases treated in this work.

BPS solutions considered in [21] (for the minimal theory). There, a number of curvature invariants were examined and were found to remain finite at $r = 0$, hinting at a regular solution. It would be interesting to study this in more detail.

4.2 A non-BPS two centre solution

We now turn to the construction of a two centre example described by the almost BPS partner of a supersymmetric black ring in Taub-NUT. Inspired by [41], we dimensionally reduce this solution to obtain a two centre system in four dimensions with a non-BPS black hole carrying $D4-D2-D0$ charges at one centre and a naked singularity carrying $D6$ charge at the other. As in the BPS case, one can hide the naked singularity by adding the electric black hole of the previous section at the centre of the geometry, so that the solution becomes regular. It is worth mentioning that our solutions seem to be related to the ones recently constructed in [44, 45]. In the latter it was argued that stationary non-BPS solutions similar to the ones in [41] should exist. It would be interesting to study this connection further.

Starting with the simple ring solution, we consider the set of harmonic functions:

$$\begin{aligned} K^I &= \frac{2}{h^0}\frac{p^I}{|\mathbf{r}-\mathbf{r}_0|}, & L_I &= l_I + \frac{2}{3}\frac{q_I}{|\mathbf{r}-\mathbf{r}_0|}, \\ M &= -b + \frac{3}{2}\frac{l_I p^I}{|\mathbf{r}-\mathbf{r}_0|}, & H &= h^0 + \frac{p^0}{r}, \end{aligned} \tag{4.9}$$

where $\mathbf{r}_0 = (0, 0, R)$. The pole in the function M is constrained by the requirement of cancelling Dirac-Misner strings near asymptotic infinity through (3.13). We omitted the constant terms in the K^I for simplicity, as they do not add important features to the solution.

Unfortunately, the conditions (3.12) and (3.13) cannot be solved explicitly in the two centre case. Thus, the solution is only known implicitly through the first order equations (3.12), (3.13) that govern the metric and the scalars and is unique given the boundary conditions at infinity implied by (4.9). This brings two complications in our analysis. First, we don't have complete control of global issues like closed time-like curves (CTC's) or Dirac strings in the full geometry. We will assume that there are no such obstructions for this solution, as for its BPS partner. In the following we verify that there are no such problems at least asymptotically and near the horizon, by restricting the charges and moduli accordingly. It appears that by requiring the conditions

$$e^{-4U} \geq 1\,, \qquad \hat{\omega}^i \hat{\omega}_i < e^{-4U}\,, \tag{4.10}$$

where the function e^{-4U} was defined in (2.26), one can eliminate these problems for the four dimensional solution that interests us most. As e^{-4U} is proportional to $g_{\psi\psi}$ in the full geometry, these constraints are also natural from a five dimensional point of view.

Another implication of the difficulty in solving for the full flow is that we are not able to find directly the relation of the distance R to the charges and moduli, since this would be obtained by an integrability condition on (3.13) after (3.12) and (3.14) are solved. Thus we will be forced to work indirectly, comparing with known attractor formulae in four dimensions to determine this parameter.

Here, we will again concentrate on the dimensionally reduced solution, for which (3.12)–(3.14) describe a first order split attractor flow for all quantities and especially the scalars, as in the BPS case. Explicitly, the metric of the four dimensional solution is as in (2.26), whereas the scalars (and their asymptotic values) and field strengths are (assume that the scalars take values in a symmetric space for simplicity):

$$z^I = -\frac{f}{H} X^I \left(H\,\omega_5 - \mathrm{i} e^{-2U}\right) + \frac{1}{2}K^I \quad \Rightarrow \quad z^I_\infty \equiv x^I + \mathrm{i}\, y^I = \frac{b\, l^I}{h^0\, l^3}\,(1+\mathrm{i}) \;, \tag{4.11}$$

$$\begin{aligned} F^I_{(4)} &= -d\left[\left(X^I f(1+H^2\omega_5^2\, e^{4U}) + \frac{1}{2}K^I H^2 \omega_5 e^{4U}\right)(dt+\hat\omega)\right] \\ &\quad + \frac{1}{4}\left(K^I \nabla_k H - H \nabla_k K^I\right)\varepsilon_{ijk} dx^i \wedge dx^j \;, \end{aligned} \tag{4.12}$$

and $F^0 = dA^0$, with A^0 as in (2.27). Note that the asymptotic scalars are as in the single centre case, but they can be generalised by adding constant terms in the harmonic functions K^I.

We now discuss the behaviour of the solution near infinity and the horizon. This will enable us to calculate the mass from asymptotic data and the entropy through the horizon area of the black ring. Consider first the limit $r \to \infty$, so that all the harmonic functions can be approximated by powers of $1/r$. Then, the solutions of (3.12),(3.14) up to first order in that expansion are:

$$f^{-1} X_I \simeq L_I \quad \Rightarrow \quad H\omega_5 \simeq \frac{3}{2}\frac{l_I p^I}{r} + M = -b + \frac{3\, l_I p^I}{r}, \tag{4.13}$$

$$\lim_{r\to\infty} e^{-2U} = \sqrt{h^0 l^3 - b^2} = 1 \tag{4.14}$$

and the asymptotic solution reduces to the one in the single centre case with the addition of magnetic charges. Note that we imposed the same constraint on the moduli. We will use the definitions in (4.5) in the following. The mass of the four dimensional two centre solution is found again by expanding e^{-2U}, with the result:

$$\begin{aligned} M_{ADM} &= \frac{1}{4G_4}\left(l^3 p^0 + 2\, h^0\, l^I q_I + 6b l_I p^I\right) \\ &= \frac{\sqrt{1+b^2}}{4G_4}\left((1+b^2) y^{3/2} p^0 + \frac{2\, y^I q_I}{y^{3/2}} + \frac{6b}{y^{3/2}} y_I p^I\right) \;, \end{aligned} \tag{4.15}$$

where we wrote everything in terms of the asymptotic moduli and we defined $y_I = \frac{1}{6} C_{IJK}\, y^J\, y^K$. We observe again that it is a marginal sum of charges, as expected, implying that there is no binding energy between the constituents of the system. We will have more to say on this when we consider the distance between the two centres.

Following [41], we also compute the ADM momentum along the fifth direction, identified as electric Kaluza-Klein charge after dimensional reduction:

$$\begin{aligned} P_{ADM} &= -\frac{1}{16\pi\, G_5}\int d\bar\psi\, d\Omega_2\, r^2\, \partial_r\, h_{\bar t \bar\psi} \\ &= \frac{\pi}{G_5}\left[-\frac{2\,\tilde b}{l}\, l^I q_I + \frac{\tilde b^2}{h^0} p^0 - \frac{3}{\sqrt{1+b^2}}\, l_I p^I \left(1 + \tilde b^2\left(1 + \frac{1}{2} l^2\right)\right)\right] \;, \end{aligned} \tag{4.16}$$

where $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ and the $\bar{t}\,,\bar{\psi}$ coordinates are related to $t\,,\psi$ by a Lorentz boost, so that $\lim_{r\to\infty} g_{\bar{t}\bar{\psi}} = \eta_{\bar{t}\bar{\psi}}$, as in [41]. We also set $\tilde{b} \equiv b/\sqrt{h^0 l^3} = b/\sqrt{1+b^2}$, in view of (4.14). In four dimensions, the charge associated to the A^0 gauge field differs from this result, as it is defined through the electric-magnetic dual of the corresponding field strength, as in (2.24). The charges of our four dimensional solution, as computed from (2.24)–(2.25) are:

$$q_A = \left(-\frac{3\,l_I p^I}{2h^0}\,, -q_I\right)\,, \qquad p^A = (p^0\,, p^I)\,, \tag{4.17}$$

where the $q_I\,, p^I$ are the ones appearing in (4.9). Note that the charge q_0 is proportional to the pole of the function M, as would be naively expected.

As a final remark on the asymptotic properties of this solution, it is important to emphasise that the electric charges in five dimensions are not equal to the four dimensional ones, as has been observed in the BPS case [22, 31, 43]. As can be easily checked, the five dimensional electric charges are shifted with respect to the q_I by a factor proportional to $b\,C_{IJK} l^J p^K$ coming from the Chern-Simons term. It would be interesting to analyse the implications of this inconspicuous moduli dependent shift on both the macroscopic supergravity solutions and the microscopic counting of the entropy for the black ring.

Now, we turn to the near horizon region of the ring, taking the limit $\mathbf{r} \to \mathbf{r_0}$. Fortunately, the near horizon region is identical to the one of the BPS solution, as explained above, so that most results of [22, 41] carry over by simply replacing $p^I \to H_0\, p^I$, where $H_0 = H(\mathbf{r_0})$. In particular, the metric of a cross section of the horizon is:

$$ds^2_{\text{hor}} = L^2 d\psi^2 + \left(\frac{H_0}{h^0}\right)^2 p^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)\,, \tag{4.18}$$

where $0 \leq \theta \leq \pi$, $0 \leq \phi \leq 2\pi$ are spherical coordinates on the ring and $0 \leq \psi \leq 4\pi$ is parametrising the S^1 of Taub-NUT along which the ring is wound. The constants p, L are given by:

$$p^3 = \frac{1}{6} C_{IJK} p^I p^J p^K\,, \qquad L^2 = \frac{2\,(h^0)^2}{H_0^2\, p}\left(D^{IJ}\, q_I\, q_J + q_0\right)\,, \qquad D^{IJ}\, C_{JKL} p^L = \delta^I_K\,, \tag{4.19}$$

with q_0 as in (4.17). Requiring that $L^2 > 0$ eliminates CTC's near the horizon, as in [41].

The entropy is found through the area of the horizon:

$$S = \frac{A}{4\,G_5} = \frac{\pi}{G_4} L \left(\frac{H_0}{h^0}\right)^2 p^2 = \frac{H_0}{h^0}\frac{2\pi}{G_4}\sqrt{\frac{p^3}{2}\left(D^{IJ}\, q_I\, q_J + q_0\right)}\,, \tag{4.20}$$

where we use four dimensional quantities. This expression agrees up to the prefactor with standard entropy formulae for the prepotential (2.19) for both BPS and non-BPS attractors obtained by analytic continuation of charges in four dimensions. As can be easily seen from (4.11), the same holds for the scalars and gauge fields with exactly the same prefactor. We use this to fix the parameter R, comparing[6] with the expressions in [7]. We find that:

$$\frac{H_0}{h^0} \equiv 1 + \frac{p^0}{h^0 R} = 1\,, \quad \Rightarrow \quad R \to \infty\,, \tag{4.21}$$

[6] Note that we use the conventions of [46].

which appears puzzling at first sight. Recalling that one finds the same result from (2.16) for the distance between two centres carrying mutually local charges in the BPS case, it suggests that such a divergence simply signals that the distance is not fixed, but arbitrary. In other words, the near horizon limit imposes $R \to \infty$, as this quantity is not fixed in terms of the charges that have a definite scaling as one takes the limit. We view this as more evidence towards the marginality of the system, consistent with the mass formula derived above. It remains an open question to solve (3.12)–(3.14) (numerically or otherwise) and verify this result in the full solution.

A related observation is that the non-BPS attractor carrying all possible charges in four dimensions could be unstable, hinting towards the existence of a multi-centred solution with the same total charge and stable components [7, 15, 47]. Our solution (more precisely the one described below, where both centres are proper attractors) is indeed a resolution of such a system. Moreover, the marginality of its constituents seems to indicate that the single centre attractor would be stable as well, unless this limit lies outside the boundary of moduli space of these solutions. This brings us back to the discussion in the end of section 4.1, since all harmonic functions would be sourced at one centre in this case. It remains an interesting open question to study these single centre geometries both in five and four dimensions and investigate whether they are proper attractors.

This completes our discussion of the non-BPS black ring and we now briefly sketch how to obtain a regular solution in four dimensions. As mentioned above, reducing a Taub-NUT solution to four dimensions produces a naked singularity carrying Kaluza-Klein magnetic charge p^0, even though the full five dimensional solution is smooth. One can construct a regular solution by hiding the naked singularity behind the horizon of a black hole placed at the centre of the five dimensional geometry, as in [41]. This is easily done through the following modification on the electric harmonic functions:

$$L_I = l_I + \frac{2}{3}\frac{q_I^{(h)}}{r} + \frac{2}{3}\frac{q_I}{|\mathbf{r}-\mathbf{r}_0|}\,, \qquad M = -b + \left(\frac{3}{2}l_A p^A - q_0^{(r)}\right)\frac{1}{r} + \frac{q_0^{(r)}}{|\mathbf{r}-\mathbf{r}_0|}\,. \tag{4.22}$$

This turns the four dimensional naked singularity into a rotating version of the black hole in the previous section, so that both centres are regular black holes with finite area horizons. Note that now there is an extra $D0$ charge present, unlike in previous examples. It can be assigned to any of the two centres, with the constraint that the total $D0$ charge is as in the simple ring solution, due to the requirement of cancelling Dirac-Misner strings asymptotically.

All the above results remain valid after this modification, by simply replacing q_0 by $q_0^{(r)}$ in the entropy formula (4.20) for the ring and q_I by $q_I + q_I^{(h)}$ in all other expressions. The total electric charge q_0 is the same as in (4.17). In particular, the mass formula (4.15) remains marginal, implying that there is no interaction energy between the two centres even after adding extra charges.

We end this section with a few comments on possible generalisations. Comparing with the most general two centre solution in four dimensions, we see that our regular solution has half of the charges turned on at one centre, all but one charges at the other and asymptotic moduli as in (4.11). Restricting to symmetric scalar manifolds, we see that this can be dualised to arbitrary moduli and charges at one centre, as in the single centre solution above. We are then only one charge short of the most general two centre solution. Finally, it should be straightforward to construct multi-black rings along the lines of [22]. These solutions would reduce to a collection of magnetically charged black holes in four dimensions.

5 Conclusion and discussion

In this work, we have identified a systematic map from BPS to non-BPS extremal black holes in four dimensional $\mathcal{N} = 2$ supergravity with cubic prepotential, that generalises the one known through the study of attractors. In doing this, we found that a simple algorithm can be obtained in the five dimensional uplift of the theory, by exploiting the properties of an extra set of BPS solutions available for a flat base space. These solutions are invariant under a different SU(2) isometry than the one seen in four dimensions and are invisible in the four dimensional BPS spectrum. By gluing together such local BPS solutions with flat base, we constructed solutions with general Gibbons-Hawking base space that preserve half of the supersymmetry locally but not globally, nicknamed almost BPS. After dimensional reduction, the result is a class of non-BPS solutions very similar to the supersymmetric solutions that have been studied extensively. In particular, all quantities in these solutions satisfy first order equations, so that a number of important techniques developed for the supersymmetric case can be applied. We have demonstrated this by constructing multi centred solutions through arbitrary harmonic functions, following similar work on BPS solutions [21, 22].

Our class of almost BPS solutions is as large as the BPS class, in the sense that it involves the same number of harmonic functions, as described below (3.14). It includes the non-BPS solutions obtained through analytic continuation of charges [7, 15, 47] and some recent generalisations [18, 19]. Thus, it is bigger than the set of non-BPS solutions known to date (up to U-duality), but smaller than the full class of non-BPS solutions. As explained in section 3.2, it only includes solutions very closely related to BPS solutions, representing special points in the four dimensional non-BPS duality orbits. However, in cases with enough symmetry, one can use our special solutions as seeds to generate the full orbit of non-BPS solutions, as in [20].

The defining property of our solutions, that they preserve four supercharges locally, was also used to clarify previous observations on special properties of four dimensional attractors [32, 37] and solutions [19]. As both the asymptotic region and the near horizon geometry of extremal black holes can be written as a timelike fibration over flat space, our five dimensional solutions are BPS in these regions. This crucial property implies a simple marginal mass formula for all such solutions, as in section 4, and simplifies near horizon considerations. In this special setting, we have analysed a known single centre example and a new two centre solution in four dimensions that represents a resolution of a possibly unstable non-BPS attractor [15, 47]. In both cases the mass formula suggests a marginally bound state, and this conclusion is strengthened by the fact that the distance between the two centres in the second case appears to be arbitrary at the level of our approximation.

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There are several directions to be investigated in the context of our solutions. First, it seems very likely that our almost BPS conditions (3.12)–(3.14) can be rewritten in terms of the symplectic section $(Y^A,\ F_A)$, that is the more natural notation in four dimensions. This could shed more light on the systematics of these solutions from a four dimensional perspective and perhaps help towards a plausible generalisation of this construction to arbitrary prepotentials. Moreover, such a formulation would make the properties of our solutions under four dimensional electric magnetic duality manifest [46, 48, 49], simplifying the construction of new solutions through dualities. Such a purely four dimensional investigation is also required to identify exactly what subspace of non-BPS solutions is spanned by the almost BPS solutions and their duals.

More importantly, it would be very interesting to investigate whether the marginality property of our examples is generic or not. Thus, one could distinguish the cases where a nontrivial moduli space of this restricted class of solutions exists, if any. The general lore is that the intricate moduli spaces of the BPS solutions should not be present, but there could still be interesting questions to be asked for a trivial moduli space, for example what happens at its boundary. A related, but equally challenging task would be to study the properties of the split attractor flows described by the almost BPS conditions. Such a program appears to require heavy use of numerical methods, in view of the extra complications compared to the BPS case. In fact, it should be noted that such studies could be problematic in view of the incompleteness of our set of solutions relative to the full non-BPS spectrum.

Finally, it would be interesting to check the robustness of our trick against higher derivative corrections. The most crucial property of the two derivative theory we used is the hyper-Kähler structure of the base. Recently, the R^2 corrected five dimensional theory was considered in [50–52], where some explicit solutions were constructed. These were found to be timelike fibrations over hyper-Kähler spaces, so that a possible extension of our analytic continuation to this theory is conceivable. Such an extension could allow the construction of non-BPS solutions in four dimensional $\mathcal{N} = 2$ supergravity with higher derivatives [12, 46, 53, 54]. We hope to return to some of these issues in the future.

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A The equations of motion

In this appendix, we outline how both expressions in (3.1)–(3.4) solve the equations of motion. The equations of motion found by varying the action (2.1) with respect to the metric, gauge fields and the n_v-1 independent scalars χ^a obtained by solving the constraint in (2.2), respectively can be written as:

$$
\begin{aligned}
R_{\mu\nu} + Q_{IJ}F^I{}_{\mu\lambda}F^J{}_\nu{}^\lambda + Q_{IJ}\partial_\mu X^I\partial_\nu X^J - \frac{1}{6}g_{\mu\nu}Q_{IJ}F^I{}_{\kappa\lambda}F^{J\kappa\lambda} &= 0\,, \\
d\left(Q_{IJ}\star F^J\right) + \frac{1}{4}C_{IJK}F^J\wedge F^K &= 0\,, \\
\left[d(\star dX_I) + \left(X_JX^LC_{IKL} - \frac{1}{6}C_{IJK}\right)(F^J\wedge\star F^K + dX^J\wedge\star dX^K)\right]\frac{\partial X^I}{\partial\chi^a} &= 0\,,
\end{aligned}
\tag{A.1}
$$

where $\star$ is the Hodge dual in five dimensions. Here and in the following, we use the identities:

$$
Q_{IJ}X^J = \frac{3}{2}X_I\,, \qquad Q_{IJ}\,\frac{\partial X^J}{\partial\chi^a} = -\frac{3}{2}\frac{\partial X_I}{\partial\chi^a}\,, \tag{A.2}
$$

which are implied by (2.2)–(2.3).

We consider a metric ansatz as in (3.1) and we take the gauge fields to be of the form

$$
F^I = \pm\, d(fX^I)\wedge(dt+\omega) \pm X^IG + \Lambda^I \tag{A.3}
$$

where, for the moment, the Λ^I are arbitrary closed two-forms on the hyper-Kähler base and as before, $G = f d\omega$. It is also convenient to define

$$
J^I = \pm X^IG + \Lambda^I. \tag{A.4}
$$

The trace-reversed Einstein equations then take the form:

$$
\begin{aligned}
X^I\Delta(f^{-1}X_I) - \frac{1}{4}\left(G^2 - \frac{2}{3}Q_{IJ}J^I\cdot J^J\right) &= 0\,, \\
f\,\nabla^qG_{pq} + \nabla^qf\,\left(2G_{pq} - 3\,X_IJ^I_{pq}\right) + 3\,\nabla^qX_I\,J^I_{pq} &= 0 \\
\frac{1}{2}\left[X^I\Delta(f^{-1}X_I) - \frac{1}{4}\left(G^2 - \frac{2}{3}Q_{IJ}J^I\cdot J^J\right)\right]\delta_{pq} - G^+_{pr}G^{-r}_q + Q_{IJ}\,J^{I+}_{pc}J^{J-c}_q &= 0\,,
\end{aligned}
\tag{A.5}
$$

where the identity

$$
A^{\pm pr}\,B^{\pm q}_r + A^{\pm qr}\,B^{\pm p}_r = \frac{1}{2}\delta^{pq}\,A^{\pm rt}\,B^{\pm}_{rt} \equiv \frac{1}{2}\delta^{pq}\,A^{\pm}\cdot B^{\pm} \tag{A.6}
$$

was used. Here, Latin letters are used for flat indices on the base space and the superscript $\pm$ denotes (anti-)selfdual forms on it. In order to solve these equations, we choose

$$
X_I\Lambda^I = \mp\frac{2}{3}G^{\pm}\,, \tag{A.7}
$$

as in (3.3). Taking a derivative and using that the Λ^I are closed leads to:

$$
\nabla^qG_{pq} = \mp f^{-1}(\tilde{*}G)_{pq}\nabla^qf - 3\,(\tilde{*}\Lambda^I)_{pq}\nabla^qX_I\,, \tag{A.8}
$$

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where $\tilde{*}$ denotes the Hodge dual on the base. It is then easy to show that the second and third equations in (A.5) are satisfied if the first one is.

Similarly, the Maxwell equation in (A.1) implies the two equations:

$$\pm\frac{3}{2}\,d\tilde{*}d(f^{-1}X_I)+3\,X_I G\wedge\left(G^{\pm}+\frac{3}{2}X_I\tilde{*}\Lambda^I\right)$$
$$\pm C_{IJK}X^K\,G\wedge\Lambda^{I\mp}+\frac{1}{4}C_{IJK}\Lambda^J\wedge\Lambda^K=0\,, \tag{A.9}$$
$$2\,f^{-1}df\wedge\left(\frac{1}{2}X_I G\pm\frac{1}{3}C_{IJK}X^K\Lambda^{\mp J}\right)+3\,X_I dX_J\wedge\tilde{*}\Lambda^J$$
$$+2\,f^{-1}d(fX_I)\wedge\left(G^{\pm}+\frac{3}{2}X_I\tilde{*}\Lambda^I\right)\pm X_I d\tilde{*}G=0\,. \tag{A.10}$$

When the above choices for the Λ^I are imposed, a bit of algebra shows that the first reduces to:

$$d\,\tilde{*}\,d(f^{-1}X_I)=\mp\frac{1}{6}C_{IJK}\Lambda^J\wedge\Lambda^K\,, \tag{A.11}$$

which can be shown to imply the 00 component of the Einstein equation above, while the second is identically satisfied by using (A.8).

Finally, the scalar equation in (A.1) reduces to:

$$\Bigg[\Delta(f^{-1}X_I)\pm\frac{1}{3}X^P C_{NPI}G\cdot\Lambda^N \tag{A.12}$$
$$+\frac{1}{2}\left(X_J X^L C_{IKL}-\frac{1}{6}C_{IJK}\right)\Lambda^J\cdot\Lambda^K\Bigg]\frac{\partial X^I}{\partial\chi^a}=0\,.$$

For the above Λ^I, the expression in brackets leads again to (A.11).

References

[1] S. Ferrara, R. Kallosh and A. Strominger, *$N = 2$ extremal black holes*, *Phys. Rev.* **D 52** (1995) 5412 [hep-th/9508072] [SPIRES].

[2] A. Strominger, *Macroscopic entropy of $N = 2$ extremal black holes*, *Phys. Lett.* **B 383** (1996) 39 [hep-th/9602111] [SPIRES].

[3] S. Ferrara and R. Kallosh, *Supersymmetry and attractors*, *Phys. Rev.* **D 54** (1996) 1514 [hep-th/9602136] [SPIRES].

[4] S. Ferrara, G.W. Gibbons and R. Kallosh, *Black holes and critical points in moduli space*, *Nucl. Phys.* **B 500** (1997) 75 [hep-th/9702103] [SPIRES].

[5] K. Goldstein, N. Iizuka, R.P. Jena and S.P. Trivedi, *Non-supersymmetric attractors*, *Phys. Rev.* **D 72** (2005) 124021 [hep-th/0507096] [SPIRES].

[6] A. Sen, *Black hole entropy function and the attractor mechanism in higher derivative gravity*, *JHEP* **09** (2005) 038 [hep-th/0506177] [SPIRES].

[7] P.K. Tripathy and S.P. Trivedi, *Non-supersymmetric attractors in string theory*, *JHEP* **03** (2006) 022 [hep-th/0511117] [SPIRES].

[8] W.A. Sabra, *General static $N = 2$ black holes*, *Mod. Phys. Lett.* **A 12** (1997) 2585 [hep-th/9703101] [SPIRES].

JHEP05(2009)058

[9] W.A. Sabra, *Black holes in $N = 2$ supergravity theories and harmonic functions*, *Nucl. Phys.* **B 510** (1998) 247 [hep-th/9704147] [SPIRES].

[10] K. Behrndt, D. Lüst and W.A. Sabra, *Stationary solutions of $N = 2$ supergravity*, *Nucl. Phys.* **B 510** (1998) 264 [hep-th/9705169] [SPIRES].

[11] P. Breitenlohner, D. Maison and G.W. Gibbons, *Four-dimensional black holes from Kaluza-Klein theories*, *Commun. Math. Phys.* **120** (1988) 295 [SPIRES].

[12] G. Lopes Cardoso, B. de Wit, J. Kappeli and T. Mohaupt, *Examples of stationary BPS solutions in $N = 2$ supergravity theories with R^2-interactions*, *Fortsch. Phys.* **49** (2001) 557 [hep-th/0012232] [SPIRES].

[13] G.W. Moore, *Arithmetic and attractors*, hep-th/9807087 [SPIRES].

[14] F. Denef, *Supergravity flows and D-brane stability*, *JHEP* **08** (2000) 050 [hep-th/0005049] [SPIRES].

[15] R. Kallosh, N. Sivanandam and M. Soroush, *Exact attractive non-BPS STU black holes*, *Phys. Rev.* **D 74** (2006) 065008 [hep-th/0606263] [SPIRES].

[16] A. Ceresole and G. Dall'Agata, *Flow equations for non-BPS extremal black holes*, *JHEP* **03** (2007) 110 [hep-th/0702088] [SPIRES].

[17] L. Andrianopoli, R. D'Auria, E. Orazi and M. Trigiante, *First order description of black holes in moduli space*, *JHEP* **11** (2007) 032 [arXiv:0706.0712] [SPIRES].

[18] G. Lopes Cardoso, A. Ceresole, G. Dall'Agata, J.M. Oberreuter and J. Perz, *First-order flow equations for extremal black holes in very special geometry*, *JHEP* **10** (2007) 063 [arXiv:0706.3373] [SPIRES].

[19] E.G. Gimon, F. Larsen and J. Simon, *Black holes in supergravity: the non-BPS branch*, *JHEP* **01** (2008) 040 [arXiv:0710.4967] [SPIRES].

[20] S. Bellucci, S. Ferrara, A. Marrani and A. Yeranyan, *STU black holes unveiled*, arXiv:0807.3503 [SPIRES].

[21] J.P. Gauntlett, J.B. Gutowski, C.M. Hull, S. Pakis and H.S. Reall, *All supersymmetric solutions of minimal supergravity in five dimensions*, *Class. Quant. Grav.* **20** (2003) 4587 [hep-th/0209114] [SPIRES].

[22] J.P. Gauntlett and J.B. Gutowski, *General concentric black rings*, *Phys. Rev.* **D 71** (2005) 045002 [hep-th/0408122] [SPIRES].

[23] S. Ferrara and M. Günaydin, *Orbits of exceptional groups, duality and BPS states in string theory*, *Int. J. Mod. Phys.* **A 13** (1998) 2075 [hep-th/9708025] [SPIRES].

[24] S. Ferrara and M. Günaydin, *Orbits and attractors for $N = 2$ Maxwell-Einstein supergravity theories in five dimensions*, *Nucl. Phys.* **B 759** (2006) 1 [hep-th/0606108] [SPIRES].

[25] S. Bellucci, S. Ferrara, M. Günaydin and A. Marrani, *Charge orbits of symmetric special geometries and attractors*, *Int. J. Mod. Phys.* **A 21** (2006) 5043 [hep-th/0606209] [SPIRES].

[26] S. Ferrara and A. Marrani, *On the moduli space of non-BPS attractors for $N = 2$ symmetric manifolds*, *Phys. Lett.* **B 652** (2007) 111 [arXiv:0706.1667] [SPIRES].

[27] L. Andrianopoli, S. Ferrara, A. Marrani and M. Trigiante, *Non-BPS attractors in 5D and 6D extended supergravity*, *Nucl. Phys.* **B 795** (2008) 428 [arXiv:0709.3488] [SPIRES].

[28] M. Gunaydin, G. Sierra and P.K. Townsend, *The geometry of $N = 2$ Maxwell-Einstein supergravity and Jordan algebras*, Nucl. Phys. **B 242** (1984) 244 [SPIRES].

[29] G.W. Gibbons and P.J. Ruback, *The hidden symmetries of multicenter metrics*, Commun. Math. Phys. **115** (1988) 267 [SPIRES].

[30] G.W. Gibbons and S.W. Hawking, *Gravitational multi-instantons*, Phys. Lett. **B 78** (1978) 430 [SPIRES].

[31] K. Behrndt, G. Lopes Cardoso and S. Mahapatra, *Exploring the relation between 4D and 5D BPS solutions*, Nucl. Phys. **B 732** (2006) 200 [hep-th/0506251] [SPIRES].

[32] G.L. Cardoso, J.M. Oberreuter and J. Perz, *Entropy function for rotating extremal black holes in very special geometry*, JHEP **05** (2007) 025 [hep-th/0701176] [SPIRES].

[33] B. de Wit and A. Van Proeyen, *Potentials and symmetries of general gauged $N = 2$ supergravity: Yang-Mills models*, Nucl. Phys. **B 245** (1984) 89 [SPIRES].

[34] B. de Wit, P.G. Lauwers and A. Van Proeyen, *Lagrangians of $N = 2$ supergravity-matter systems*, Nucl. Phys. **B 255** (1985) 569 [SPIRES].

[35] B.E.W. Nilsson and C.N. Pope, *Hopf fibration of eleven-dimensional supergravity*, Class. Quant. Grav. **1** (1984) 499 [SPIRES].

[36] M.J. Duff, H. Lü and C.N. Pope, *Supersymmetry without supersymmetry*, Phys. Lett. **B 409** (1997) 136 [hep-th/9704186] [SPIRES].

[37] A. Ceresole, S. Ferrara and A. Marrani, *4D/5D correspondence for the black hole potential and its critical points*, Class. Quant. Grav. **24** (2007) 5651 [arXiv:0707.0964] [SPIRES].

[38] D. Gaiotto, A. Strominger and X. Yin, *New connections between 4D and 5D black holes*, JHEP **02** (2006) 024 [hep-th/0503217] [SPIRES].

[39] D. Gaiotto, A. Strominger and X. Yin, *5D black rings and 4D black holes*, JHEP **02** (2006) 023 [hep-th/0504126] [SPIRES].

[40] J.C. Breckenridge, R.C. Myers, A.W. Peet and C. Vafa, *D-branes and spinning black holes*, Phys. Lett. **B 391** (1997) 93 [hep-th/9602065] [SPIRES].

[41] H. Elvang, R. Emparan, D. Mateos and H.S. Reall, *Supersymmetric 4D rotating black holes from 5D black rings*, JHEP **08** (2005) 042 [hep-th/0504125] [SPIRES].

[42] D. Gaiotto, W.W. Li and M. Padi, *Non-supersymmetric attractor flow in symmetric spaces*, JHEP **12** (2007) 093 [arXiv:0710.1638] [SPIRES].

[43] I. Bena, P. Kraus and N.P. Warner, *Black rings in Taub-NUT*, Phys. Rev. **D 72** (2005) 084019 [hep-th/0504142] [SPIRES].

[44] K.-M. Lee, E.O. Colgain, H. Yavartanoo and K.P. Yogendran, *BPS D0-D6 branes in supergravity*, JHEP **03** (2009) 009 [arXiv:0811.2086] [SPIRES].

[45] J. Camps, R. Emparan, P. Figueras, S. Giusto and A. Saxena, *Black rings in Taub-NUT and D0-D6 interactions*, JHEP **02** (2009) 021 [arXiv:0811.2088] [SPIRES].

[46] T. Mohaupt, *Black hole entropy, special geometry and strings*, Fortsch. Phys. **49** (2001) 3 [hep-th/0007195] [SPIRES].

[47] S. Nampuri, P.K. Tripathy and S.P. Trivedi, *On the stability of non-supersymmetric attractors in string theory*, JHEP **08** (2007) 054 [arXiv:0705.4554] [SPIRES].

[48] B. de Wit, *Electric-magnetic duality in supergravity*, *Nucl. Phys. Proc. Suppl.* **101** (2001) 154 [hep-th/0103086] [SPIRES].

[49] P. Aschieri, S. Ferrara and B. Zumino, *Duality rotations in nonlinear electrodynamics and in extended supergravity*, *Riv. Nuovo Cim.* **31** (2009) 625 [arXiv:0807.4039] [SPIRES].

[50] A. Castro, J.L. Davis, P. Kraus and F. Larsen, *5D black holes and strings with higher derivatives*, *JHEP* **06** (2007) 007 [hep-th/0703087] [SPIRES].

[51] A. Castro, J.L. Davis, P. Kraus and F. Larsen, *Precision entropy of spinning black holes*, *JHEP* **09** (2007) 003 [arXiv:0705.1847] [SPIRES].

[52] A. Castro, J.L. Davis, P. Kraus and F. Larsen, *String theory effects on five-dimensional black hole physics*, *Int. J. Mod. Phys.* **A 23** (2008) 613 [arXiv:0801.1863] [SPIRES].

[53] G. Lopes Cardoso, B. de Wit, J. Kappeli and T. Mohaupt, *Stationary BPS solutions in $N = 2$ supergravity with R^2 interactions*, *JHEP* **12** (2000) 019 [hep-th/0009234] [SPIRES].

[54] B. Sahoo and A. Sen, *Higher derivative corrections to non-supersymmetric extremal black holes in $N = 2$ supergravity*, *JHEP* **09** (2006) 029 [hep-th/0603149] [SPIRES].